

Improving SAT Solvers via Blocked Clause Decomposition

Jingchao Chen

School of Informatics, Donghua University
2999 North Renmin Road, Songjiang District, Shanghai 201620, P. R. China
`chen-jc@dhu.edu.cn`

Abstract. The decision variable selection policy used by the most competitive CDCL (Conflict-Driven Clause Learning) SAT solvers is either VSIDS (Variable State Independent Decaying Sum) or its variants such as exponential version EVSIDS. The common characteristic of VSIDS and its variants is to make use of statistical information in the solving process, but ignore structure information of the problem. For this reason, this paper modifies the decision variable selection policy, and presents a SAT solving technique based on BCD (Blocked Clause Decomposition). Its basic idea is that a part of decision variables are selected by VSIDS heuristic, while another part of decision variables are selected by blocked sets that are obtained by BCD. Compared with the existing BCD-based technique, our technique is simple, and need not to reencode CNF formulas. SAT solvers for certified UNSAT track can apply also our BCD-based technique. Our experiments on application benchmarks demonstrate that the new variables selection policy based on BCD can increase the performance of SAT solvers such as abcdSAT. The solver with BCD solved an instance from the SAT Race 2015 that was not solved by any solver so far. This shows that in some cases, the heuristic based on structure information is more efficient than that based on statistical information.

Keywords: CDCL SAT solver, Blocked Clause Decomposition, Decision variable selection policy

1 Introduction

The SAT solving technology has made great progress in recent years. A number of state-of-the-art SAT solvers have come to the fore. Nevertheless, a great number of SAT problems remain unsolved yet. How to improve further SAT solvers is still a very important topic.

Recently, Balyo et al [3] reported that reencoding CNF (Conjunctive Normal Form) formulas by BCD (Blocked Clause Decomposition) can improve the performance of the state-of-the-art CDCL (Conflict-Driven Clause Learning) SAT solvers such as Lingeling [6]. The basic idea behind this technique is to define multiple versions for each variable through reencoding the original CNF formula. The version number of variables depends on blocked subsets obtained by a decomposition algorithm. A blocked set is defined to be a set of clauses that can

be removed completely by BCE (Blocked Clause Elimination) [10]. It is easy to verify that any CNF formula can be decomposed into two blocked subsets. Due to this property, any CNF formula can be reencoded into a new CNF formula in the order where clauses occur in blocked subsets. According to the experiments of Balyo et al [3], some reencoded application benchmarks are indeed easier to be solved than the original ones. For a part of hard application benchmarks, the reencoding starts to pay off after 3500 seconds. The main drawback of the reencoding technique is that UNSAT problems solved by it cannot be certified, since there is no automatic method for recognizing whether the original benchmark and the reencoded benchmark are identical or not. Therefore, the BCD-based reencoding is not suitable for SAT solvers that are used for the certified UNSAT track.

This paper aims at improving the performance of SAT solvers by BCD. In general, a CDCL SAT solver consists of components such as decision variable selection, Boolean constraint propagation, learnt clause database reduction, restart etc. This paper focuses how to improve the decision variable selection policy of CDCL solvers. Unlike the reencoding approach by Balyo et al [3], we do not reencode CNF formulas, but use blocked sets to guide the selection of a few decision variables. Up to now, the decision variable selection policy used by the most competitive CDCL SAT solvers is based on either VSIDS (Variable State Independent Decaying Sum) [11] or its variants such as EVSIDS (exponential VSIDS) [9,5], VMTF (variable move-to-front), ACIDS (average conflict-index decision score) [4]. The common characteristic of VSIDS and its variants is to make use of statistical information in the solving process, but ignore structural information of the problem. The empirical evaluation of Biere et al [4] shows that EVSIDS, VMTF, and ACIDS empirically perform equally well. Therefore, we believe that improving the performance of SAT solvers by only modifying VSIDS leads difficultly to a breakthrough. For this reason, we decide to use structure information obtained by BCD to optimize the decision variable selection policy. Our basic idea is that a part of the decision variables are selected by VSIDS heuristic, while another part of the decision variables are selected by blocked subsets that are obtained by BCD. Solver *abcdSAT* [8], which is built on the top of Glucose 2.3 [1,2], is the first to optimize the variable selection policy with BCD, and won the Gold Medal of the main track of SAT Race 2015 [12]. From the result of SAT Race 2015, the BCD-based variable selection policy improved indeed the performance of this solver. This paper identifies further such an evaluation. The original *abcdSAT* applied the BCD-based technique only for small instances, not for large instances. Thus, at SAT Race 2015, no solver solved a large instance *group_mulr*. However, if the BCD-based technique is used also for large instances in the initial phase of search, *group_mulr* can be solved in 105.2 seconds by *abcdSAT*. Our BCD-based technique need not reencoding. Its advantage is that solvers entering certified UNSAT track can apply directly it also.

2 Preliminaries

In this section, after the definition of some notations, we introduce the basic principle of a modern CDCL SAT solver, on which the improvement will be made in Section 4.

A formula in CNF is defined as a conjunction of clauses, where each clause is a disjunction of literals, each literal being either a Boolean variable or its negation. Usually, the logic form of a clause C is written as $C = x_1 \vee \cdots \vee x_m$, where $x_i (1 \leq i \leq m)$ is a literal. A clause with only one literal is called a unit clause or unit literal. A CNF formula F is written as $F = C_1 \wedge \cdots \wedge C_n$, where $C_i (1 \leq i \leq n)$ is a clause.

Given two clauses $C_1 = l \vee a_1 \vee \cdots \vee a_m$ and $C_2 = \bar{l} \vee b_1 \vee \cdots \vee b_n$, the clause $C = a_1 \vee \cdots \vee a_m \vee b_1 \vee \cdots \vee b_n$ is called the resolvent of C_1 and C_2 on the literal l , which is denoted by $C = C_1 \otimes_l C_2$.

The so-called blocked clause can be defined formally as follows. Given a CNF formula F , a clause C , a literal $l \in C$ is said to block C w.r.t. F if (i) C is a tautology w.r.t. l , or (ii) for each clause $C' \in F$ with $\bar{l} \in C'$, the resolvent $C' \otimes_l C$ is a tautology. When l blocks C w.r.t. F , the literal l and the clause C are called a blocking literal and a blocked clause, respectively.

In general, a CDCL SAT solver consists of unit propagation, variable activity based heuristic, literal polarity phase, clause learning, restarts and a learnt clause database reduction policy etc. Here is the core framework of a CDCL SAT solver.

Algorithm CDCL_solver

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repeat the following steps
  if  $\text{propagate}()$  then
    if  $(c = \text{conflictAnalyze}()) == \emptyset$  then return UNSAT
    add  $c$  to learnt clause database
    backtrack to the assertion level of  $c$ 
  else if  $(l = \text{pickBranchLit}()) == \text{null}$  then return SAT
    assert literal  $l$  in a new decision level

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In the above algorithm, Procedure *propagate* performs unit propagation, i.e., assigns repeatedly each unit literal to true until the formula F has no unit clause under the current model. When this procedure yields a conflict, a new asserting clause c is derived by Procedure *conflictAnalyze*. If the derived clause c is empty, then the formula F is unsatisfiable. Otherwise, it is added to the learnt clause database, and the algorithm backtracks to the assertion level of the learnt clause c , i.e., the level where the learnt clause becomes unit. If unit propagation does not generate the empty clause, Procedure *pickBranchLit* begins to select a new decision literal l . If such a literal is selected successfully, it is asserted in a new decision level. Otherwise, the formula is answered to be satisfiable. Throughout this paper, Procedure *pickBranchLit* is assumed to use the EVSIDS heuristic to select a literal with the highest score.

3 Related Work

In theory, any CNF formula can be decomposed into two blocked subsets such that both can be solved by BCE (Blocked Clause Elimination). Therefore, we can assume that one blocked set of a CNF formula is $C_1 \wedge C_2 \wedge \dots \wedge C_m$, where C_i ($i = 1, 2, \dots, m$) is a clause. Balyo et al [3] reencode each clause C_i so that the reencoded formula is solved more easily than the original formula. Their reencoding may be described as follows. Each blocking literal x_i is allowed to have several versions. In the order of $C_m, C_{m-1} \dots, C_1$, each of its versions is defined. Assuming that clause C_t has the form of $C_t = x_i \vee y_{j_1} \vee \dots \vee y_{j_k}$, where x_i is the blocking literal. Let $x_{i,\$}$ be the current version of x_i . Its next version is defined as follows.

$$x_{i,\$+1} := x_{i,\$} \vee (y_{j_1,\$} \wedge \dots \wedge y_{j_k,\$})$$

Then this formula is converted to CNF. Clearly, since a blocking literal is mapped to multiple version variables, such a reencoding technique will add a vast amount of auxiliary variables. As far as we know, so far no solver entering SAT competition (Race) used such a reencoding technique.

4 Decision Variable Selection Policy Based on Blocked Clause Decomposition

In this section we describe a new SAT solving technique, which is based on BCD.

Apart from the reencoding technique of Balyo et al, to avoid using auxiliary variables, our BCD-based solving policy does not reencode the original CNF formula, but uses blocked subsets to guide the selection of decision variables. The reencoding technique of Balyo et al is to how to convert a CNF formula into one which is easily solved, but does not modify any SAT solver. Nevertheless, our BCD-based solving technique is to modify a SAT solver. Assuming that a CNF formula is decomposed into a large blocked subset L and a small blocked subset S . The reencoding technique uses each blocked subset separately, while we use them by appending S to L . For convenience, let $L \wedge S = C_1 \wedge C_2 \wedge \dots \wedge C_n$, where C_i ($i = 1, 2, \dots, n$) is a clause, and the order of clauses in L and S is opposite to that in which clauses are eliminated by BCE. We use the locations where variables occur in $L \wedge S$ for the first time to determine decision variable selection at some levels in the pickBranchLit procedure of a CDCL solver. Let $pos[v]$ denote the minimum index where variable v occur in $C_1 \wedge C_2 \wedge \dots \wedge C_n$. Ignoring the priority of clauses, the position indexes of variables may be computed as follows.

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for each variable  $v$  do  $pos[v] = 0$ 
for  $i = 1$  to  $n$  do
    for each variable  $v \in C_i$  do
        if  $pos[v] = 0$  then  $pos[v] = i$ 

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In our real implementation, the priority of binary clauses is higher than that of the other clauses. That is to say, if there exist non-binary clause C_j and binary clause C_k that both contain v , and there is no binary clause C_i ($i < k$) containing v , $pos[v]$ is set to k even if $j < k$. Hence, the exact expression of $pos[v]$ may be described as follows.

$$pos(v) = \begin{cases} \arg \min\{v \in C_i \wedge |C_i| = 2\} & \exists_i |C_i| = 2 \\ \arg \min\{v \in C_i\} & \forall_i |C_i| \neq 2 \\ 0 & \text{otherwise} \end{cases}$$

Using $pos[v]$, procedure *pickBranchLit* of CDCL SAT solvers may be modified as follows.

Algorithm pickBranchLit
if current level $\in \{1, 2, 3\} \wedge \#conf < \theta$ **then**
 Let v be decision variable at 0 level
 $S \leftarrow \emptyset$
 for $i = pos[v]$ to $pos[v] + 5$ **do**
 if C_i is satisfied **then continue**
 $S \leftarrow S \cup C_i$
 if $S \neq \emptyset$ **then return** literal $l \in S$ with EVSID-based highest score
return literal $l \in F$ with EVSID-based highest score

At whichever decision level, the algorithm selects always a literal l with the highest score computed by EVSID. At the 1st – 3rd decision level, the selection range is limited to S , which is a subset of formula F , while at the other levels, it is F , not subset S , i.e., there is no limitation. According to our experiments, it is a good choice that only those three levels adopt the BCD-based decision variable selection policy. In the above pseudo-code, $\#conf$ denote the number of conflicts. In general, θ is set to 30000 for large instances, and 500000 for small instances. We use condition $\#conf < \theta$ to limit the application range of the BCD-based policy. When selecting a decision variable at the 1st – 3rd level, we consider at most 6 clauses C_i ($pos[v] \leq i \leq pos[v] + 5$) in the order of the blocked clauses, where v is a decision variable at the 0-th level. Among these candidate clauses, we pick a literal with the EVSID-based highest score as a decision literal. Whenever a part of variables are fixed, our solver runs a simplification procedure. In general, after the simplification procedure, we can obtain a blocked set that is different from the initial blocked set. However, to save the cost of computing repeatedly the blocked set, we do not update $pos[v]$. In other words, what we used is the initial $pos[v]$ (blocked set), not the updated $pos[v]$. From this viewpoint, our BCD-based policy is static, not dynamic.

5 Empirical evaluation

We evaluated the performance of SAT solving with BCD and without BCD under the following experimental platform: Intel Core 2 Quad Q6600 CPU with speed of 2.40GHz and 2GB memory.

Table 1. Runtime of solver abcdSAT with different modes on application benchmarks that were not solved by at least one out of four modes: no BCD, BCD1-3. $|F|$ is in thousands of clauses. Time is in seconds.

Instances	$ F $	$\#var$	S/ U	abcdSAT no BCD	abcdSAT BCD1	abcdSAT BCD2	abcdSAT BCD3
korf-18	207	7178	U	> 5000	420.5	579.8	420.5
group_mulr	4302	1052071	U	> 5000	> 5000	105.2	105.2
52bits_12.dimacs	19	872	S	> 5000	> 5000	4940.5	4940.5
aes_32_3_keyfind_1	2	397	S	180.1	> 5000	4324.8	4324.8
aes_64_1_keyfind_1	2	276	S	> 5000	846.7	2457.7	2457.7
grieu-vmpe-31	104	961	S	1566.6	4966.0	> 5000	1566.6
gss-22-s100	52	9330	S	> 5000	812.6	> 5000	812.6
lgiraldezlevy.2200.9086.08.40.2	11	1998	S	> 5000	608.9	4148.6	608.9
lgiraldezlevy.2200.9086.08.40.62	11	2048	S	> 5000	829.7	1303.2	829.7
lgiraldezlevy.2200.9086.08.40.83	11	1988	S	2038.1	> 5000	> 5000	> 5000
lgiraldezlevy.2200.9086.08.40.93	12	2039	S	> 5000	4055.7	864.2	4055.7
manthey_DimacsSorter_35_8	52	3349	S	> 5000	2841.8	3591.8	2841.8
manthey_DimacsSorterHalf_35_8	52	3349	S	> 5000	2729.1	3448.7	2729.1
manthey_DimacsSorter_37_3	57	3492	S	426.1	> 5000	> 5000	> 5000
manthey_DimacsSorterHalf_37_3	57	3492	S	439.2	> 5000	> 5000	> 5000
mrpp_8x8#22_24	118	10097	S	> 5000	4378.1	> 5000	4378.1
manthey_DimacsSorterHalf_37_9	70	4594	S	2824.5	> 5000	> 5000	> 5000

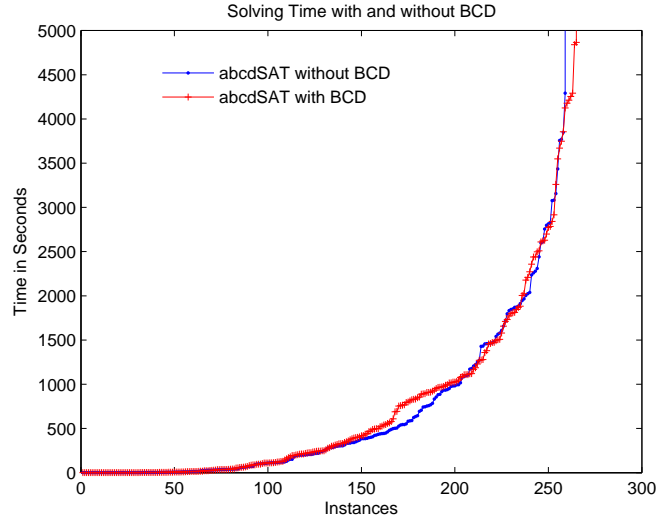


Fig. 1. The number of instances that abcdSAT with and without BCD can solve in a given amount of time. The x- and y-axis denote the number of solved instances and running time in seconds, respectively. The time limit for each instance was 5000 seconds.

In this experiment, to verify the efficiency of the BCD-based policy, we use solver abcdSAT [8], the winner of the main track of SAT Race 2015, which is built on the top of the CDCL solver Glucose 2.3. BCD algorithm used is a recently proposed algorithm called *MixDecompose* [7], The source code of *MixDecompose* is available at <http://github.com/jingchaochen/MixBcd>. As an advantage, *MixDecompose* can ensure that the decomposition of any instance is done within 200 seconds. And its decomposition quality keeps high still. Here, the decomposition quality is measured by $\frac{|L|}{|F|}$, where $|L|$ and $|F|$ denote the number of clauses in the large blocked set and the original formula, respectively.

Table 1 shows the runtime of abcdSAT with four different modes (versions). The timeout for each solver to solve each instance was set to 5000 seconds. Mode *no BCD* denotes that abcdSAT does not use any BCD-based policy. All the other three modes are based on BCD. The difference among these three modes is that the values of θ in condition $\#conf < \theta$ in algorithm *pickBranchLit* are different. Let n and m denotes the number of clauses and variables in formula F , respectively. θ is set as follows.

Mode *BCD1*:

$$\theta = \begin{cases} 0 & n > 15 \times 10^5 \vee m > 5 \times 10^5 \\ 6 \times 10^6 & \text{otherwise} \end{cases}$$

Mode *BCD2*:

$$\theta = \begin{cases} 0 & n > 5 \times 10^6 \vee m > 15 \times 10^5 \vee n < 2m \\ 30000 & \text{above is false} \wedge m > 5 \times 10^5 \\ 5 \times 10^5 & \text{otherwise} \end{cases}$$

Mode *BCD3*:

$$\theta = \begin{cases} 0 & n > 5 \times 10^6 \vee m > 15 \times 10^5 \vee n < 2m \vee n > 30m \\ 30000 & \text{above is false} \wedge m > 5 \times 10^5 \\ 6 \times 10^6 & \text{above is false} \wedge m \geq 1600 \wedge m \leq 15000 \\ 5 \times 10^5 & \text{otherwise} \end{cases}$$

Mode *BCD1* is actually the SAT-Race 2015's version of abcdSAT.

In Table 1, Column $|F|$ denotes the number of clauses in formula F in thousands of clauses. Here F is a formula simplified by the preprocessing of abcdSAT, not the original input formula. $\#var$ denotes the number of variables in F . Column *S/U* indicates whether an instance is SAT or UNSAT. Table 1 lists all the application benchmarks from SAT Race 2015 where the performance of four modes is inconsistent, except the first one from SAT Competition 2014. For benchmarks that are not listed in Table 1, either all the four modes solved them or no mode solved them in 5000 seconds. As seen in Table 1, the solvers with BCD are better than the solver without BCD. The performance of Mode *BCD3* is the best. It solved 7 more instances than the mode without BCD. Mode *BCD3* solved *group_mulr* in 105.2 seconds. This instance was not solved by any

solver in the SAT Race 2015. Notice, the SAT-Race 2015’s version of abcdSAT adopted the BCD policy only for small instances, not for large instances such as *group_mulr*. In addition, no MiniSat-style solver solved *korf-18*. Our BCD-based version solved easily it.

Figure 1 shows a cactus plot related to the comparison of abcdSAT with and without BCD. In this comparison experiment, all 300 instances tested are from the main track at the SAT Race 2015. Here the BCD mode used is Mode *BCD3*. As seen in the cactus plot, when the given amount of time is small, the solver with BCD has no advantage. However, when it is plenty large enough, the solver with BCD solved more instances than the solver without BCD.

6 Conclusions

In this paper, we presented a BCD-based improvement strategy on SAT solvers, which is different from that of Balyo et al [3]. Common to the two strategies is that they start to pay off after sufficiently long time. If a given amount of time is very short, there is no improvement on SAT solvers. Compared with the approach of Balyo et al, our approach is simple, need not reencoding, and has no application limit. In addition, the BCD-based reencoding of Balyo et al is a preprocessing, while our BCD-based mode is a solving strategy.

The decision variable selection is a very important component of CDCL SAT solvers. Our BCD-based variable selection policy solved an instance that was not solved by any solvers so far. However, it seems to be suited only for a part of instances. What is the optimal variable selection policy after all? This is a problem which is well worth looking into further. The current version of our BCD-based strategy is static. However, in the usual sense, dynamic is better than static. It is left as an open problem how to make the BCD-based strategy dynamic.

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